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THE EFFECTS OF AN INNOVATION: A TRADE THEORY APPROACH

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1. Introduction

The question we wish to address in this paper is: what are the economic effects due to the introduction of a new technology on the private production sector of a small open economy.

Thus our paper is basically concerned with modelling the effects of technological change. "Traditional" approaches to modelling the effects of technological change in a small open economy are reviewed and extended in Jones (1979, pp. 14-17). In these approaches, technological change is regarded as an exogenous change in the input-output coefficients of existing industries, or the new technology is embodied in new units of the capital stock. Our approach will be somewhat different. We shall follow in the footsteps of Rodriguez (1975), who regarded a new technology as simply the existence of a new production sector in a standard two good, two factor, two country model of international trade. Our model will be less general than that of Rodriguez in that it will be a small country model that holds world prices fixed, but it will be more general in that we allow for arbitrary numbers of internationally traded goods, domestic goods and industries. Furthermore, Rodriguez did not analyze the effects on factor prices of the introduction of the new technology, a topic that we address in sections 4 and 6 below.

What are the benefits to the country under consideration of allowing the new technology to be introduced? We assume that the benefit is the extra national product (valued at the constant world prices) that the country can produce with the help of the new technology, assuming that the country's endowment of resources and factors remains fixed. Thus we have a measurement problem that is analogous to the classical gains from trade problem: what is the gain that a small economy facing fixed world prices can make moving from an autarky (no trade) equilibrium to a free trade equilibrium?

In section 2, we lay out our model of the private production sector of a small open economy. The model is due to Diewert and Woodland (1977) and Jones and Scheinkman (1977) and it generalizes the standard no joint production model that appears in the international trade literature (e.g., see Samuelson (1953) and Dixit and Norman (1980)).

In section 3, we introduce the innovation into the economy. The innovation is modelled as the sudden appearance of a new sector whose technology is characterized by a set of input-output coefficients. These input-output coefficients have the property that they generate a positive profit m* when priced out at the pre innovation producer prices. We assume that a monopolist introduces the new technology into the economy at some markup m between 0 (this corresponds to the limiting case of a competitive post innovation economy) and m* (this corresponds to the limiting case of the competitive pre innovation economy). Given the markup m, the economy's post innovation equilibrium can be characterized by the solution to a concave programming problem. We obtain several dual representations of this problem (which prive to be useful in later sections) by applying the Karlin (1959) - Uzawa (1958) Saddle Point Theorem.

In section 4, we show that the economy cannot lose by the introduction of the new technology and that the benefits to the economy are maximized if the technology is introduced in a competitive manner (the m=0 case). Our proofs do not involve any restrictive differentiability assumptions; they follow simply by looking at the structure of the constrained maximization problems that characterize the alternative equilibria. Thus our proofs are based on the revealed preference techniques popularized by Samuelson (1947) in the consumer context and Debreu (1959) in the producer context.

In section 5, we supplement the general approach of section 4 with an approach to measuring the gains from introducing the new technology that relies on differentiability assumptions. An advantage of the differentiable approach is that we obtain approximate formulae for the gain that enable one to form quantitative (rather that qualitative) estimates for the gain that depend on potentially observable elasticities.[1]

In section 6, we maintain our differentiability assumptions but we turn our attention to the effects of the new technology on factor markets.

Our differentiability assumptions are also maintained in sections 7 and 8 where we allow the government to attempt to offset the monopoly power of the holder of the new technology. Section 7 considers the case of a domestic monopolist while section 8 considers the case of a foreign monopolist. The type of problem analyzed in this section has been studied a great deal in the multinational enterprise literature.[2] We follow in this tradition and in section 8, we assume that the new technology has been developed abroad (by a multinational firm say) and hence any monopoly profits are remitted abroad. Our analysis here complements that of Katrak (1977)(1981) and Svedberg (1979) who developed models where a multinational firm behaves monopolistically.

Section 9 concludes. A Mathematical Appendix contains proofs for various propositions stated in the text.

2. A Model of the Production Sector of an Open Economy

Consider the profit maximizing part of the production sector of a small open economy. We assume that there are M commodities that are traded internationally at constant prices $(p_1,p_2,\ldots,p_M)^T \equiv p \gg 0_M$[3]. We assume that there are N domestic commodities (e.g., primary inputs, domestic services, natural resources) and K constant returns to scale industries or firms.[4] Sector k has access to a technologically feasible set of net outputs[5] of internationally traded and domestic goods S^k, for $k=1,\ldots,K$. Since each sector exhibits constant returns to scale, the sets S^k are cones and have the following representations: for $k=1,\ldots,K$,

$$S^k \equiv \{(u^k, -v^k)\colon u^k = z_k y^k,\ v^k = z_k x^k,\ z_k \geq 0,\ (y^k, -x^k) \in C^k\}$$

where $z_k \geq 0$ is a nonnegative sector k <u>scale</u> variable and C^k is a closed convex set of feasible sector k unit scale net output vectors. Each set C^k could consist of just a single set of input-output coefficients. In this case, our model of the production sector reduces to the usual activity analysis model (e.g., see Dorfman, Samuelson and Solow (1958, pp.130-185)). On the other hand, we could have the number of internationally traded goods M equal to K, the number of sectors, with $C^k = \{(e_k, -x^k)\colon f^k(x^k) \geq 1,\ x^k \geq 0_N\}$ where e_k is a unit vector with a one in component k and f^k is the production function for sector k. This case corresponds to the usual neoclassical no joint production model that occurs in the pure theory of international trade.[6] In this case, we can identify z_k, the scale of sector k, with the output produced by sector k. In the general case, y^k is a unit scale net output vector of internationally traded goods for sector k and x^k is a unit scale net input vector of domestic goods for sector k. Normally trade theorists assume that all domestic goods are inputs into every sector so that $x^k \geq 0_N$ for every k, and we will use this assumption in section 6 below.

For price vectors $p \gg 0_M$, $w \gg 0_N$, define the sector k <u>unit</u> (scale) <u>profit function</u> π^k for $k = 1,2,\ldots,K$ by

(1) $\pi^k(p,w) \equiv \max_{y,x} \{p \cdot y - w \cdot x : (y, -x) \in C^k\}.$

It can be shown (e.g., see Diewert (1974) or Diewert and Woodland (1977)) that the unit profit functions π^k are linearly homogeneous and convex functions in their price arguments p and w.

Let $(y^{k*}, -x^{k*})$ be the sector k input-output coefficient vector that is observed in the pre innovation economy for $k = 1,2,\ldots,K$, and let $z_k^* > 0$ be the corresponding observed scale in sector k. Define the economy's (net) domestic input vector v[7] by

(2) $v \equiv \sum_{k=1}^K x^{k*} z_k^*.$

We assume that the pre innovation production sector of the economy behaves competitively; more specifically, we assume that the observed input output coefficients, y^{k*}, x^{k*}, and the observed scales z_k^* solve the following constrained maximization problem (3), which is the problem of maximizing the net value of output, valued at the international price vector p, subject to the N domestic resource contraints:

(3) $$G^* \equiv \max_{y^k, x^k, z^k} \{ \sum_{k=1}^K p \cdot y^k z_k : \sum_{k=1}^K x^k z_k \leq v \; ; \; z_k \geq 0, \quad (y^k, -x^k) \in C^k, \; k = 1,\ldots,K\}.$$

The constrained maximization problem (3) is a variable coefficients programming problem (see Dantzig (1965, pp. 433-447)); if the sets C^k are single points (the activity analysis case), then (3) reduces to an ordinary linear programming problem.

It will prove to be useful to obtain some alternative expressions for (3). We may introduce a vector of nonnegative Lagrange (or Kuhn-Tucker) multipliers $w \geq 0_N$ which corresponds to the domestic resource constraints $\Sigma_k x^k z_k \leq v$ in (3) and apply the Karlin (1959, p. 201) - Uzawa (1958, p. 34) Saddle Point Theorem[8] to the primal problem (3) and we obtain the following dual expression for G*:

$$(4) \quad G^* = \max_{y^k, x^k, z_k} \min_{w \geq 0_N} \{ \textstyle\sum_{k=1}^K (p \cdot y^k - w \cdot x^k) z_k + w \cdot v : z_k \geq 0,\ (y^k, -x^k) \in C^k,\ k=1,\ldots,K \}$$

$$(5) \quad = \max_{z \geq 0_K} \min_{w \geq 0_N} \{ \textstyle\sum_{k=1}^K \pi^k(p,w) z_k + w \cdot v : z \equiv (z_1, \ldots, z_K)^T \}$$

$$= \textstyle\sum_{k=1}^K \pi^k(p, w^*) z_k^* + w^* \cdot v$$

where $z^* \equiv (z_1^*, \ldots, z_K^*)^T$ and $w^* \equiv (w_1^*, \ldots, w_N^*)^T$ solve the max-min problem (5). Formula (5) follows from the line above by using definitions (1). The vector z* may be interpreted as a vector of optimal industry scales and w* may be interpreted as a vector of optimal domestic good prices. We assume $w^* \gg 0_N$.

It is possible to apply the Karlin-Uzawa Saddle Point Theorem "in reverse" to (5) in order to obtain a constrained minimization problem involving only the domestic price variables w.[9] The resulting "pure" price dual to (3) is:

$$(6) \quad G^* = \min_{w \geq 0_N} \{ w \cdot v : \pi^k(p,w) \leq 0,\ k = 1,2,\ldots,K \}.$$

We note that z_k^* may be interpreted as the optimal Lagrange multiplier that corresponds to the k^{th} constraint in (6).

Thus we have obtained four alternative expressions, (3)-(6), for the maximal value of output expressed in world prices. A solution to the primal problem (3) yields the optimal input-output coefficients y^{k*}, x^{k*} for the economy and the corresponding optimal sectoral scales z_k*, while a solution $w*$ to the dual problem (6) yields optimal domestic prices for the economy. If the sets C^k are single points (the activity analysis case), then the nonlinear programming problems (3) and (6) reduce to linear programs where (6) is the dual to the primal problem (3).

We are now ready to introduce an innovation into the economy.

3. The Post Innovation Economy

Suppose a new scientific, engineering or managerial discovery is made which leads to the possibility of producing an existing good at a lower cost. We follow the example of Rodriguez (1975) by assuming that the discovery may be modelled as the sudden appearance of a new sector, sector K+1 say, in the list of profit maximizing sectors in our small open economy. We could assume that the new technology is widely known, so that competition forces profits down to zero in the new industry or we could consider the case of a closely held new technology where there is the possibility of making monopoly profits. Our task in the present section is to develop a framework that will enable us to compare the prediscovery structure of production and prices with the corresponding post-discovery structure, irrespective of whether the new technology is introduced in a competitive or noncompetitive manner.

We assume that the new technology can be represented by a single set of input-output coefficients, $y^{K+1}, -x^{K+1}$, so that the set C^{K+1} consists of a single point; i.e.,

$$(7) \quad C^{K+1} \equiv \{(y^{K+1}, -x^{K+1})\}.$$

Recall that the vector of international prices is $p \gg 0_M$ and that the pre innovation vector of domestic prices was $w^* \gg 0_N$. An essential feature of the new technology is that when it is run at unit scale, it makes a positive profit, say $m^* > 0$. let π^{K+1} be the unit profit function for the new technology. Our assumptions imply

$$(8) \quad \pi^{K+1}(p,w^*) \equiv p\cdot y^{K+1} - w^*\cdot x^{K+1} = m^* > 0.$$

Consider the case of a monopolist who holds the rights to the new technology. The monopolist could decide to impose any markup m between 0 and m* per unit of scale. However, once the markup m is chosen, the scale of operations $z_{K+1}(m)$ of the monopolist becomes an endogenous variable. We can determine $z_{K+1}(m)$ (and the other scales $z_k(m)$ of the competitive sectors) by solving the following constrained maximization problem which is analogous to (3):

$$(9) \quad Y(m) \equiv \max_{y^k,x^k,z_k}\{\textstyle\sum_{k=1}^K p\cdot y^k z_k + (p\cdot y^{K+1}-m)z \quad : \sum_{k=1}^{K+1} x^k z_k \le v \; ;$$
$$(y^k,-x^k)\varepsilon C^k,\ k=1,\ldots,K \; ; \; z \equiv (z_1,\ldots,z_K,z_{K+1})^T \ge 0_{K+1}\}.$$

Note that when m=0 (so that the monopolist imposes no markup and thus behaves competitively), (9) reduces to the post innovation competitive output maximization problem; i.e., the maximization problem (9) with m=0 is the analogue to the competitive maximization problem (3), where we have added the new sector K+1 to the list of industries.

Recall that we applied the Karlin-Uzawa Saddle Point theorem to the constrained maximization problem (3) to obtain the max-min problem (5) and the constrained minimization problem (6). We may perform analogous applications to the constrained maximization problem (9) to obtain the max-min problem (10) and the constrained minimization problem (11):

$$(10)\quad Y(m) = \max_{z \geq 0_{K+1}} \min_{w \geq 0_N} \{w \cdot v + \textstyle\sum_{k=1}^{K} \pi^k(p,w)z_k + (p \cdot y^{K+1} - w \cdot x^{K+1} - m)z_{K+1} :$$

$$z \equiv (z_1,\ldots,z_K,z_{K+1})^T \geq 0_{K+1}\}$$

$$(11)\quad = \min_{w \geq 0_N} \{w \cdot v : \pi^k(p,w) \leq 0,\ k=1,\ldots,K\ ;\ p \cdot y^{K+1} - w \cdot x^{K+1} - m \leq 0\}.$$

Note that problem (11) is the problem of choosing domestic prices $w \equiv (w_1,\ldots,w_N)^T$ to minimize net factor income $w \cdot v$ subject to each competitive industry making zero or negative profits and subject to the new industry making unit scale profits $p \cdot y^{K+1} - w \cdot x^{K+1}$ equal to or less than the given markup m.[10] Let z(m), w(m) solve the max-min problem (10). Then the Karlin-Uzawa theorem also implies that $z(m) \equiv [z_1(m),\ldots,z_K(m), z_{K+1}(m)]^T$ is a z solution for (9) and w(m) is a solution to the constrained minimization problem (11) and z(m) is a corresponding vector of optimal Kuhn-Tucker multipliers for the constraints in (11). Furthermore, z(m) and w(m) satisfy the following complementary slackness conditions:

$$(12)\quad \pi^k(p,w(m))z_k(m) = 0\ ,\ k=1,\ldots,K\ ;$$

$$(13)\quad (p \cdot y^{K+1} - w(m) \cdot x^{K+1} - m)z_{K+1}(m) = 0.$$

Using the equivalence of (9)-(11) and the complementary slackness conditions (12) and (13), it can be seen that if the monopolist chooses the markup m between 0 and m*, then the monopolist's optimal scale $z_{K+1}(m)$ is the largest z_{K+1} solution to (9) or (10). This choice of z_{K+1} will lead to maximal overall profits for the monopolist consistent with: (i) competitive profit maximizing behavior or the part of sectors 1 to K, (ii) satisfaction of the domestic resource constraints and (iii) total monopoly profits divided by the monopolist's optimal scale being equal to or less than the prespecified unit scale markup m. Thus solutions to the constrained maximization problem (9) provide a convenient method for generating an equilibrium allocation of resources within the production sector of the post innovation economy given that the monopolist has decided on a unit scale markup of size m where $0 \leq m < m^*$. The case m=0 corresponds to the competitive introduction of the new technology. In all cases, Y(m) is the equilibrium net factor invome (excluding any monopoly profits from factor income). Equivalently, Y(m) is the net value of internationally traded goods valued at world prices minus any pure profits accruing to the monopolist.

What happens if the monopolist sets his markup equal to the initial cost advantage m* of the new technology? The following proposition gives the answer.

Propostion 1: If the monopolist sets his markup equal to m*, the initial cost advantage of the new technology, then the post innovation value of factor income will equal the pre innovation value of factor income; i.e., $Y(m^*) = G^*$ where Y is defined by (9) and G* is defined by (3). Furthermore, if $y^{1*},\ldots,y^{K*},x^{1*},\ldots,x^{K*}$, $z_1^*,\ldots,z_K^*$ solves (3) and w* solves (6), then $y^{1*},\ldots,y^{K*},x^{1*},\ldots,x^{K*}$, $z_1^*,\ldots,z_K^*$, $z_{K+1}^* \equiv 0$ solves (9) with m=m* and w* solves (11) when m=m*; i.e., the pre innovation equilibrium input-output coefficients, sectoral scales and domestic prices are also post innovation equilibrium quantities and prices if the monopolist (foolishly) chooses his markup to be the initial cost advantage m*.

The domestic net factor income function Y(m) defined by (9), (10) or (11) for markups m between 0 and m* provides a convenient summary statistic of the effect of introducting the innovation into the economy in a monopolistic or competitive manner. Propostion 1 shows that Y(m*) is the pre innovation level of domestic net factor income G^* and we indicated earlier that $Y(0)$ is the post innovation level of domestic net factor income if the innovation is introduced into the economy in a competitive manner.

It is also useful to define the economy's net output (of internationally traded goods) function $G(m)$ as a function of the monopolist's markup m. For $0 \leq m < m^*$, define $z_{K+1}(m)$ as the largest z_{K+1} solution to the constrained maximization problem (9). Define $z_{K+1}(m^*)=0$. Then for $0 \leq m \leq m^*$,

$$(14) \quad G(m) \equiv Y(m) + mz_{K+1}(m).$$

Thus the economy's net output function $G(m)$ is equal to domestic net factor income $Y(m)$ plus monopoly profits $mz_{K+1}(m)$, which in turn is equal to (using (9)) the net value of internationally traded goods produced by the private production sector of the economy. Since monopoly profits equal 0 when $m=0$ and when $m=m^*$, we have

$$(15) \quad Y(m^*) = G(m^*) = G^* \quad \text{and} \quad Y(0) = G(0).$$

In the next section, we shall determine how the net factor income and net output functions, $Y(m)$ and $G(m)$, behave as the monopolistic markup m goes from m^* (which corresponds to the pre innovation economy) to 0 (which corresponds to the competitive introduction of the innovation).

4. The Gains from the New Technology: General Results

If the technology monopolist is a domestic national, then the relevant "welfare" function is the net output of internationally traded goods function, $G(m)$. If the monopolist is a foreigner, then the relevant "welfare" function is $G(m)$ minus monopoly profits which is equal to $Y(m)$, domestic net factor income.

Proposition 2: $Y(m)$ declines monotonically as the markup m increases from 0 (this corresponds to the competitive post innovation equilibrium) to the initial cost advantage of the new technology m^* (this corresponds to the competitive pre innovation equilibrium) ; i.e., for $0 \leq m' < m'' \leq m^*$, $Y(m') \geq Y(m'')$.

Proposition 3: The new sector's equilibrium scale $z_{K+1}(m)$ declines monotonically as the markup m incrases from 0 to m^*, i.e., for $0 \leq m' < m'' \leq m^*$, $z_{K+1}(m') \geq z_{K+1}(m'')$.

Proposition 4: $G(m)$ declines monotonically as the markup m increases from 0 to m^*; i.e., for $0 \leq m' < m'' \leq m^*$, $G(m') \geq G(m'')$.

Thus as the technology monopolist decreases his markup from m^* (which corresponds to the pre innovation economy where $G^* = G(m^*)$ and $z_{K+1}(m^*) = 0$) to 0 (which corresponds to the competitive adoption of the new technology), the real net output of internationally traded goods $G(m)$ grows steadily, net factor income $Y(m)$ grows steadily and the optimal scale of the new industry $z_{K+1}(m)$ grows steadily. Thus in our simplified model, the home country cannot lose if the new technology is adopted, even if all monopoly profits are repatriated abroad. This is a strong qualitative result, derived under relatively weak assumptions.

Let us now turn our attention to the behavior of domestic prices w as the markup m changes. In Propositions 5 and 6 below, let $0 \leq m' < m'' \leq m^*$, and let w' and w'' be solutions to the minimization problem (11) when $m=m'$ and m'' respectively. Thus w' and w'' are equilibrium vectors of domestic prices when the monopolist chooses markups m' and m'' respectively. The following Proposition follows directly from Proposition 2 and the equality of (9) and (11).

Proposition 5: $w' \cdot v \geq w'' \cdot v$ where $0 \leq m' < m'' \leq m^*$.

Proposition 6: $w' \cdot x^{K+1} \geq w'' \cdot x^{K+1}$ where $0 \leq m' < m'' \leq m^*$.

In the pure theory of international trade, it is often assumed taht the domestic goods are all inputs. In this case, the net endowment vector v is positive and the domestic good input coefficients are nonnegative and nonzero, i.e., $v \gg 0_N$ and $x^{K+1} > 0_N$. Thus in this trade theory case, we may divide both sides of the inequality in Propostion 5 by $w'' \cdot v > 0$ and both sides of the inequality in Propostion 6 by $w'' \cdot x^{K+1} > 0$ and we obtain the following inequalities:

(16) $w' \cdot v / w'' \cdot v \geq 1$;

(17) $w' \cdot x^{K+1} / w'' \cdot x^{K+1} \geq 1$.

Thus as the markup decreases, two fixed basket type indexes of factor prices increase; the quantity weighting vector is the aggregate endowment vector v in (16) and is the vector of domestic input-output coefficients for the new technology x^{K+1} in (17). Thus, for these two fixed weight aggregates, domestic inputs cannot lose as the new technology is introduced into the economy.

We conclude this section with a somewhat technical result that is not required in the remainder of the paper.

Proposition 7: Write the net domestic factor income function defined by (9) as Y(m,p,v). Then Y(m,p,v) is a convex function of m and p for fixed v and Y(m,p,v) is a concave, nondecreasing function of v for fixed m,p. Let $0 \leq m' \leq m^*$, $p' \gg 0_M$ and $v' \gg 0_N$ and let $y^{k'}$, $x^{k'}$, z_k' solve (9), and let w' solve (11) when m=m', p=p' and v=v'. In addition, suppose

$Y(m,p,v)$ is once differentiable with respect to its arguments at m',p',v'. Then:

(18) $\nabla_m Y(m',p',v') = -z'_{K+1}$; (minus the equilibrium scale for the new sector)

(19) $\nabla_v Y(m',p',v') = w'$; (equilibrium domestic prices)

(20) $\nabla_p Y(m',p',v') = \sum_{k=1}^{K+1} p' \cdot y^{k'} z'_k$; (Equilibrium net supplies of traded goods)

where $\nabla_m \equiv \partial Y/\partial m$, $\nabla_v Y \equiv (\partial Y/\partial v_1,\ldots,\partial Y/\partial v_N)^T$ and $\nabla_p Y \equiv (\partial Y/\partial p_1,\ldots,\partial Y/\partial p_M)^T$ denote the partial derivative of Y with respect to m and the gradient vectors of Y with respect to the components of the vectors v and p respectively.

Thus if the factor income function Y happens to be differentiable, its partial derivatives (18)-(20) yield valuable comparative statics information. In the differentiable case, the "optimal" scale for the new sector is $z_{K+1}(m,p,v) \equiv -\partial Y(m,p,v)/\partial m$, the vector of "optimal" factor prices regarded as a function of m, p and v is $w(m,p,v) \equiv \nabla_v Y(m,p,v)$ and the vector of "optimal" net supplies of internationally traded goods is $y(m,p,v) \equiv \nabla_p Y(m,p,v)$. If Y happens to be twice continuously differentiable with respect to its arguments at m',p',v', then using (18) and the convexity of Y in m, we can deduce

(21) $\partial z_{K+1}(m',p',v')/\partial m = -\partial^2 Y(m',p',v')/\partial m^2 \leq 0,$

which is the differentiable counterpart to Proposition 3. Using (20) and the convexity of Y in p, we may deduce that the M by M matrix of partial derivatives of the traded goods supply functions $y(m,p,v)$ with respect to the components of the vector of international prices p is $\nabla_p y(m',p',v') =$

$\nabla^2_{pp}Y(m',p',v')$, where $\nabla^2_{pp}Y$ is symmetric, positive semidefinite matrix of second order partial derivatives of Y with respect to the components of p. Since the main diagonal elements of a positive semidefinite matrix are nonnegative, we must have

$$(22) \quad \partial y_i(m',p',v')/\partial p_i = \partial^2 Y(m',p',v')/\partial p_i^2 \geq 0 \ , \ i=1,\ldots,M \ ;$$

i.e., the net supply functions for internationally traded goods do not slope downwards. Using (19) and the concavity of Y in v, we may deduce that the N by N matrix of partial derivatives of the domestic price functions w(m,p,v) with respect to the components of the net endowment vector v is $\nabla_v w(m',p',v') = \nabla^2_{vv} Y(m',p',v')$, where $\nabla^2_{vv}Y$ is the symmetric, negative semidefinite matrix of second order partial derivatives of Y with respect to the components of v. Thus we must have

$$(23) \quad \partial w_n(m',p',v')/\partial v_n = \partial^2 Y(m',p',v')/\partial v_n^2 \leq 0, \ n=1,\ldots,N \ ;$$

i.e., the return to the nth net factor cannot increase as the amount of that factor increases. Finally, (18)-(20) and the symmetry of the matrix of second order partial derivatives of Y(m',p',v') with respect to all of its arguments yield the following Samuelson (1953) type reciprocity relations:

$$(24) \quad -\partial z_{K+1}(m',p',v')/\partial p_i = \partial y_i(m',p',v')/\partial m \ , \quad i=1,\ldots,M \ ;$$

$$(25) \quad -\partial z_{K+1}(m',p',v')/\partial v_n = \partial w_n(m',p',v')/\partial m \ , \quad n=1,\ldots,N \ ;$$

$$(26) \quad \partial y_i(m',p',v')/\partial v_n = \partial w_n(m',p',v')/\partial p_i \ , \quad i=1,\ldots,M, \ n=1,\ldots,N.$$

Our derivation of the relationships (21)-(26) has been rather terse. More leisurely expositions of analogous results may be found in Diewert (1974, p.142-146)(1983a).

The comparative statics results on the functions $z_{K+1}(m,p,v)$, $y(m,p,v)$ and $w(m,p,v)$ which followed proposition 7 are perhaps a bit incomplete. The problem is that we may not want to regard the monopolist's markup m as being an exogenous variable. In sections 6 and 7 below, we endogenize the choice of m. However, our present comparative statics results are still useful if the new technology is introduced in a competitive manner, in which case we can set m=0 and the look at the effects of changes in p and v.

Propositions 2 and 4 show that the country gains as the new technology is introduced. However, these Propositions give us no indication as to the probable size of the gain. Thus in the following section, we make some additional regularity assumptions and derive various second order approximations to the gain from introducing the new technology in a competitive manner. These approximate gain measures depend on various potentially observable production elasticities.

5. Second Order Approximations to the Gain from Introducing the Technology

Our first additional assumption over the assumptions made in the previous section is that we assume twice continuous differentiability of the unit profit functions $\pi^k(p,w)$ whenever necessary.[11] For a given markup m such that $0 \leq m \leq m^*$, we assume that the positive domestic price vector $w(m) \gg 0_N$ solves the constrained minimization problem (11) and $z(m) > 0_{K+1}$ is the corresponding vector of Lagrange multipliers. We also assume that all K+1 constraints in (11) are binding. Then the following first order conditions must hold:

$$(27) \qquad \textstyle\sum_{k=1}^{K} \nabla_w \pi^k(p,w(m))z_k(m) - x^{K+1} z_{K+1}(m) = -v \quad ,$$

(28) $\pi^k(p,w(m)) = 0$, $k=1,\ldots,K$, and

(29) $p\cdot y^{K+1} - w(m)\cdot x^{K+1} = m.$

The equations (27)-(29) are N+K+1 equations in the N domestic prices $w(m) \equiv [w_1(m),\ldots,w_N(m)]^T$ and the K+1 industry scales $z(m) \equiv [z_1(m),\ldots,z_{K+1}(m)]^T$. After multiplying equations (27)-(29) through by -1 and differentiating with respect to the markup m, we obtain the following system of equations involving the derivative vectors $w'(m)$ and $z'(m)$:

$$
\begin{aligned}
(30)\quad -S_{ww}(m)w'(m) + X(m)z'(m) &= 0_N \\
X(m)^T\, w'(m) \qquad\qquad &= -e_{K+1}
\end{aligned}
$$

where e_{K+1} is a K+1 dimensional unit vector with a one in component K+1 and the aggregate positive semidefinite and symmetric producer substitution matrix for domestic goods S_{ww} and the N by K+1 matrix of unit scale net factor demands X are defined by

(31) $S_{ww}(m) \equiv \sum_{k=1}^{K} \nabla_{ww}^2 \pi^k(p,w(m))z_k(m)$, and

(32) $X(m) \equiv [-\nabla_w \pi^1(p,w(m)),\ldots,-\nabla_w \pi^K(p,w(m)),x^{K+1}]$

In order to invert the coefficient matrix in the left hand side of (30), it is necessary to make the following assumption:

(33) $X(m)$ has rank K+1 and $S_{ww}(m) + X(m)X(m)^T$ has rank N.

Diewert and Woodland (1977, p.391) show that assumption (33) is sufficient to imply the existence of the inverse matrix in (34):

$$(34)\quad \begin{bmatrix} -S_{ww}(m) , & X(m) \\ X(m)^T , & 0_{K+1xK+1} \end{bmatrix}^{-1} \equiv \begin{bmatrix} D(m), & E(m) \\ E(m)^T, & F(m) \end{bmatrix}$$

where D(m) is an N by N symmetric negative semidefinite matrix of rank N-K-1 (we require $K+1 \leq N$), E(m) is an N by K+1 matrix, and F(m) is a K+1 symmetric positive semidefinite matrix. Diewert and Woodland (1977, p.392) also show that each zero eigenvalue of $S_{ww}(m)$ reduces the rank of F(m) by one. An important point to notice about (34) is that a knowledge of the input-output coefficient matrix X(m) and the aggregate domestic goods substitution matrix S(m) suffices to determine the matrix F(m), which plays an important role in the subsequent analysis.

Premultiplying both sides of (30) by the inverse matrix in (34) yields the following expressions:

$$(35)\quad w'(m) = -E(m)e_{K+1} \quad ; \quad z'(m) = -F(m)e_{K+1}.$$

We may now derive Propositions 8 and 9 which are differentiable versions of Propositions 2 and 4 respectively.

<u>Proposition 8</u>: The derivative of the net domestic factor income function with respect to the markup m is equal to the negative of the equilibrium scale of the new sector and hence is nonpositive; i.e.,

$$(36)\quad Y'(m) = v\cdot w'(m) = -z_{K+1}(m) \leq 0 \quad \text{for} \quad 0 \leq m \leq m^*.$$

<u>Proposition 9</u>: The derivative of the net output of internationally traded goods function with respect to the markup m is equal to the markup times the derivative of the scale function for the new sector and hence using (35), is nonpositive; i.e.,

(37) $G'(m) = mz'_{K+1}(m) = -me^T_{K+1}F(m)e_{K+1} \leq 0$ for $0 \leq m \leq m^*$.

We may now form first order Taylor series approximations to the gains from introducing the new technology in a competitive manner. Recall that the pre innovation net factor income is $Y(0)$ and the competitive post innovation net factor income is $Y(m^*)$. Two first order approximations to the gain in factor income from introducing the new technology in a competitive manner are:

$$\begin{aligned} Y(0)-Y(m^*) &\approx Y'(m^*)(0-m^*) \\ &= m^*z_{K+1}(m^*) && \text{using (36)} \\ &= 0 && \text{since } z_{K+1}(m^*) = 0. \end{aligned} \tag{38}$$

$$\begin{aligned} Y(0)-Y(m^*) &\approx -Y'(0)(m^*-0) \\ &= m^*z_{K+1}(0) && \text{using (36).} \end{aligned} \tag{39}$$

Two first order approximations to the gain in net output from introducing the new technology in a competitive manner are:

$$\begin{aligned} G(0) - G(m^*) &\approx G'(0)(0-m^*) \\ &= 0 && \text{using (37).} \end{aligned} \tag{40}$$

$$\begin{aligned} G(0) - G(m^*) &\approx - G'(m^*)(m^*-0) \\ &= m^{*2}e^T_{K+1}F(m^*)e_{K+1} && \text{using (37).} \end{aligned} \tag{41}$$

<u>Proposition 10</u>: A quadratic approximation to the gain in factor income from introducing the new technology in a competitive manner is an average of the two first order approximations (38) and (39); i.e.,

(42) $Y(0)-Y(1) \approx (1/2)m^*z_{K+1}(0) > 0$.

Thus the approximate gain is equal to one half the initial cost advantage of the new technology m* times the post innovation competitive scale of the new technology, $z_{K+1}(0)$.

Proposition 11: A quadratic approximation to the gain in net output from introducing the new technology in a competitive manner is an average of the two first order approximations (40) and (41), i.e.,

$$(43)\quad G(0)-G(1) \approx (1/2)m^{*2}e_{K+1}^{T}F(m^{*})e_{K+1}$$

$$(44)\quad = -(1/2)m^{*2}z'_{K+1}(m^{*}) \qquad \text{using (35).}$$

Recall (15); i.e., $Y(0)-Y(1) = G(0)-G(1)$. The second order approximations to the gain from introducing the new technology, (42) and (43), look different, but (44) provides a connection. Since $z_{K+1}(m^{*}) = 0$,

$$z_{K+1}(0) = z_{K+1}(0) - z_{K+1}(m^{*}) \approx -z'_{K+1}(m^{*})(m^{*}-0)$$

and thus (42) and (43) do approximate each other.

Instead of taking averages of first order approximations to obtain second order approximations, it is possible to form second order Taylor series approximations to $Y(0)-Y(1)$ and $G(0)-G(1)$ directly. The following two Propositions result.

Proposition 12: A quadratic approximation to the gain in factor income from introducing the new technology in a competitive manner is

$$Y(0)-Y(m^*) \approx Y'(m^*)(0-m^*) + (1/2)Y''(m^*)(0-m^*)^2$$

$$= (1/2)m^{*2}Y''(m^*) \qquad \text{using (36) and } z_{K+1}(m^*) = 0$$

$$= -(1/2)m^{*2}z'_{K+1}(m^*) \qquad \text{differentiating (36)}$$

$$(45) \qquad = (1/2)m^{*2}e^T_{K+1}F(m^*)e_{K+1} \qquad \text{using (35)}$$

Proposition 13: A quadratic approximation to the gain in net output of internationally traded goods from introducing the new technology in a competitive manner is

$$G(0)-G(1) \approx -G'(0)(m^*-0) - (1/2)G''(0)(m^*-0)^2$$

$$= -(1/2)m^{*2}G''(0) \qquad \text{using (37) evaluated at } m=0$$

$$= -(1/2)m^{*2}z'_{K+1}(0) \qquad \text{differentiating (37) at } m=0$$

$$(46) \qquad = (1/2)m^{*2}e^T_{K+1}F(0)e_{K+1} \qquad \text{using (35).}$$

Note that (43) and (45) coincide. From the viewpoint of ex post analysis, the most useful expression for the approximate gain is (42). From the ex ante point of view, attempting to evaluate the potential gain from introducing the new technology in a competitive manner, the most useful expression is (43) or (45): all we need to know is the cost advantage of the new technology m^* and the matrix $F(m^*)$, which can be calculated via (34) if we know the pre innovation input-output matrix $X(m^*)$ and the pre innovation aggregate domestic goods substitution matrix $S_{ww}(m^*)$.

Formula (43) also yields some insight into the sources of the gain due to the adoption of the new technology: the gain grows quadratically as the cost advantage of the new technology m^* grows linearly, and the gain grows linearly as all elements of the domestic good substitution matrix $S_{ww}(m^*)$ grow linearly.[12] Thus the bigger is the cost advantage and the bigger are the domestic good substitution possibilities, the bigger is the gain in real output.

6. The Effects on Factor Markets

What effect on domestic prices will the introduction of the new technology have? To the first order, this question may be answered by looking at the signs of the elements of the derivative vector $w'(m^*) \equiv [w_1'(m^*),\ldots,w_N'(m^*)]^T$: if $-w_n'(m^*) < 0$, then the price of domestic good n will decrease as the new technology is introduced; i.e., as m is reduced from the pre innovation level m*.

For all of the Propositions in the remainder of this paper, we make the same differentiability and regularity assumptions as we made in the previous section, in particular, we assume (33) holds when m=m*.

Proposition 14: (i) If the new technology uses domestic goods only as inputs, then at least one domestic good will increase in price as the new technology is introduced. (ii) If the number of domestic goods exceeds one and one or more of the pre innovation industries uses domestic goods only as inputs, then at least one domestic good will increase in price and another domestic good will decrease in price as the new technology is introduced.

In order to obtain more concrete results, we now assume that N=2 and the two domestic goods are both inputs. We also assume that K=1 so that there is only one pre invention sector (sector one) and sector 2 is the new industry. The unit scale input requirements vector for sector 1 at the pre innovation equilibrium is $x^1(m^*) \equiv [x_1^1,x_2^1]^T$. We reorder the inputs if necessary so that the determinant of the $X(m^*) \equiv [x^1,x^2]$ matrix is positive; i.e., so that

$$(47)\quad |X(m^*)| \equiv |x^1,x^2| = x_1^1x_2^2 - x_2^1x_1^2 \equiv \delta > 0.$$

Proposition 15: Suppose the number of domestic goods is N=2 and both goods are inputs. Suppose that K=1 so that there is only one pre innovation sector in the economy. Suppose further that (47) holds so that the pre invention economy is input 1 intensive while the new technology (sector 2) is input 2 intensive. Then input 2 will gain as the new technology is introduced and input 1 cannot gain (and will definitely lose if the pre innovation economy uses input 2; i.e., if $x_2^1 > 0$).

If the two inputs are labour and capital and the new technology is relatively capital intensive, then capital will gain and labour will lose as the new technology is adopted. Thus this kind of technological progress acts in the same manner (with respect to the behavior of input prices) as an exogenous increase in the supply of labour relative to the supply of capital in a one sector, two input model.[13]

We now consider the case where N=3 and the three domestic goods are all inputs. We also assume that the number of sectors K+1 equals three. In this case, the matrix of domestic good input-output coefficients is $X(m^*) \equiv [x^1, x^2, x^3]$, a three by three matrix of full rank. We reorder inputs if necessary and assume that sector 1 is input 1 intensive and sector 2 is input 2 intensive, so that:

$$(49)\quad x_1^1 > 0,\ x_2^2 > 0,\ \text{and}\ x_1^1 x_2^2 - x_2^1 x_1^2 \equiv \delta > 0.$$

Proposition 16: Suppose K+1=N=3 and all domestic goods are inputs, and sector 1 is input 1 intensive, sector 2 in input 2 intensive (i.e., (48) is satisfied) and the new sector 3 is input 3 intensive (i.e., the determinant of $X(m^*) \equiv [x^1, x^2, x^3]$ is positive). Then input 3 always gains as the innovation is introduced into the economy (i.e., as the markup m is reduced from m*) and whether inputs 1 and 2 gain or lose to the first order can be determined from the following table (+ means gain, - means loss):

(49)

	Case (i)	Case (ii)	Case (iii)	Case (iv)	Case (v)	Case (vi)
$-w_1'(m^*)$	0	-	-	-	0	+
$-w_2'(m^*)$	0	+	0	-	-	-
$-w_3'(m^*)$	+	+	+	+	+	+

Case (i) assumes $x_2^1 = x_3^1 = x_3^2 = 0$. Cases (ii) to (vi) assume $x_3^1 > 0$ and the following additional assumptions: Case (ii): $x_3^2/x_3^1 < x_1^2/x_1^1$; Case (iii): $x_3^2/x_3^1 = x_1^2/x_1^1$; Case (iv): $x_1^2/x_1^1 < x_3^2/x_3^1 < x_2^2/x_2^1$; Case (v): $x_3^2/x_3^1 = x_2^2/x_2^1$; Case (vi): $x_2^2/x_2^1 < x_3^2/x_3^1$. If the new sector, sector 3, is not input 3 intensive (i.e., the determinant of the pre innovation domestic goods unit scale input requirements matrix X(m*) is negative), then we obtain Cases (vii) to (xii) which correspond to Cases (i) to (vi) in (49), except the signs of the derivatives are reversed.

Suppose input 1 is unskilled labour, input 2 is skilled labour, input 3 is capital, industry 1 is unskilled labour intensive, industry 2 is skilled labour intensive and the new industy 3 is capital intensive so that (48) is satisfied and $|X(m^*)| > 0$. Then we have x_1^2/x_1^1 = unskilled labour used in industry 2 divided by the unskilled labour used in industry 1 < x_2^2/x_2^1 = skilled labour used in industry 2 divided by the skilled labour used in industry 1 using (48). The industry 1 and 2 capital ratio is x_3^2/x_3^1 = amount of capital services used in industry 2 divided by the amount of capital services used in industry 1. If $x_3^2/x_3^1 < x_1^2/x_1^1$ so that the industry 1 and 2 capital ratio is less than the corresponding unskilled labour ratio, then we are in Case (ii), and unskilled labour loses while skilled labour and capital gain as the new technology is introduced. If $x_1^2/x_1^1 < x_3^2/x_3^1 <$ x_2^2/x_2^1 so that the industry 1 and 2 capital ratio is between corresponding unskilled and skilled labour ratios, then we are in Case (iv) and both types of labour will lose as the new technology is introduced while capital will gain. Finally if $x_2^2/x_2^1 < x_3^2/x_3^1$ so that the industry 1 and 2 capital ratio is greater than the corresponding skilled labour ratio, then we are in Case (vi) so that unskilled labour and capital gain while skilled labour loses as the new technology is introduced.

For another example where we can apply our N = K+1 = 3 model, consider the specific factor model of Jones (1979, ch.6) and apply it to the pre innovation economy. Thus let input 1 be a capital service that is specific to industry 1 and let input 2 be a capital service that is specific to industry 2 so that $x_2^1 = 0 = x_1^2$. Let industry 3 (the new industry) use the two types of capital and suppose that it uses relatively little of the third input, labour. In particular, we assume $x_1^1 > 0$, $x_2^2 > 0$, $x_3^1 > 0$ and $|X(m^*)| \equiv x_1^1 x_2^2 x_3^3 - x_1^3 x_2^2 x_3^1 - x_1^1 x_2^3 x_3^2 < 0$. With these assumption, we end up in Case (x): $|X(m^*)| < 0$, $0 = x_1^2/x_1^1 < x_3^2/x_3^1 < x_2^2/x_2^1 = +\infty$. We find that both types of capital gain as the new technology is introduced and labour loses.

The above examples show that the question about who gains and who loses as a new technology is introduced is extremely complex, even in very simple models. However, if the number of domestic goods N is equal to K+1, the number of existing industries plus the new industry, then since $-w'(m) = (X(m)^T)^{-1}e_{K+1}$, it is possible to determine which factors will gain or lose provided that we know the input-output coefficients for the existing industries and for the new industy.[14] If N > K+1, then we would also require information on substitution matrices.

Our analysis up to this point assumed that the new technology would be introduced in a competitive manner or that the monopolist's markup was determined exogenously. It is time to relax this assumption.

7. The Case of a Domestic Monopolist

Given that the monopolist has chosen the markup m, the equilibrium scale vector $z \equiv (z_1,\ldots,z_{K+1})^T$ and the equilibrium domestic good price vector $w \equiv (w_1,\ldots,w_N)^T$ are determined by equations (27)-(29). We assume that assumption (33) holds for $0 \leq m \leq m^*$ and we define the monopolist's pure profits function R(m) by

$$(50) \quad R(m) \equiv m z_{K+1}(m).$$

The monopolist's profit maximization problem is $\max_m\{R(m): 0 \leq m \leq m^*\}$. We know that $R(0) = R(m^*) = 0$ and using (35), we can compute the following derivatives:

$$(51) \quad R'(0) = z_{K+1}(0) > 0 \; ; \quad R'(m^*) = m^* z'_{K+1}(m^*) \leq 0.$$

Using (51), it can be seen using the continuity of the function R(m) that R(m) will attain a maximum over the interval $0 \leq m \leq m^*$ at some interior point $\tilde{m}$ in this interval. Since R(m) is differentiable at $\tilde{m}$, we must have

$$(52) \quad R'(\tilde{m}) = z_{K+1}(\tilde{m}) + \tilde{m} z'_{K+1}(\tilde{m}) = 0 \; ; \; 0 < \tilde{m} < m^*.$$

It should be noted that our monopoly problem is somewhat unusual, since the international prices $p \equiv (p_1,\ldots,p_M)^T$ are fixed and not affected by the monopolist's actions. However, the choice of a markup m does affect the industry scale vector z(m) and the vector of domestic prices w(m), and hence all producers find that their profit maximizing input and output decisions are affected by the monopolist's choice of m. Thus the pure profit function R(m) defined by (50) is a "general" equilibrium profit function for the monopolist and should be contrasted with the usual partial equilibrium type of profit function for a monopolist that occurs in intermediate microeconomic textbooks.

A monopolist who had full information on the technologies of the other sectors would choose the profit maximizing markup $\tilde{m}$ which satisfies (52). However, it is unlikely that a real life monopolist would have such information. The typical low information monopolist would know that his profits would be maximized for some markup between 0 and m^*, but he would not know how to compute the derivative $z'_{K+1}(m)$. Still, by a process of trial and error, the monopolist might come close to finding the $\tilde{m}$ which maximizes profits. Thus we shall assume that the profit maximizing monopolist can find $\tilde{m}$.

In the case of a domestic monopolist, we previously determined that the country's benefit function was $G(m)$ defined by (14), where G is the economy's net output of internationally traded goods function. Since the monopolist's profit maximizing $\tilde{m}$ is less than m^*, Proposition 4 shows that the country gains from the introduction of the new technology, even if the "secrets" of the new technology are closely held by a domestic monopolist.

Suppose now that the government tries to gain some of the benefits of the new technology from the monopolist. We suppose that the government places a subsidy s (if s is negative, then it is a tax) on each unit of the scale of the new sector.[15] Thus for each markup m chosen by the monopolist, he would make his monopoly profits $mz_{K+1}(m)$ plus the subsidy revenue $sz_{K+1}(m)$.

We assume that the monopolist has full information, regards the government subsidy s as a fixed number, and attempts to solve the following profit maximization problem:

(53) $\max_m R(m,s) = \max_m \{(m+s)z_{K+1}(m)\}$.

The first and second order sufficient conditions for the markup m to solve (53) are:

(54) $\partial R(m,s)/\partial m = z_{K+1}(m) + (m+s)z'_{K+1}(m) = 0$ and

(55) $\partial^2 R(m,s)/\partial m^2 = 2z'_{K+1}(m) + (m+s)z''_{K+1}(m) < 0$.

We assume that (54) and (55) hold in the following Proposition. Equation (54) implicitly determines the monopolist's markup $\tilde{m}(s)$ as a function of the government subsidy s.

Proposition 17: If the government's initial scale subsidy s is zero, then increasing s will decrease the monopolist's markup and hence (using Proposition 4) increase the government's benefit function. In order to maximize the government benefit function G(m), the government should choose the optimal subsidy $\tilde{s}$ defined by

(56) $\tilde{s} = -z_{K+1}(0)/z'_{K+1}(0) > 0$

assuming that $z'_{K+1}(0) \neq 0$; see (35) and (34) for the definition of $z'_{K+1}(0)$.

It can be verified that the equation in (56) corresponds to (54) when m is set equal to zero. The case m=0 corresponds to the competitive introduction of the new technology and yields the maximal net output of internationally traded goods (see Proposition 9).

Thus an all knowing government[16] can induce a well informed domestic monopoly holder of the rights to the new technology to produce the optimal (competitive) amount of output by dangling an appropriate subsidy before the eyes of the monopolist.

We turn now to the analysis of a foreign monopolist, where the situation is quite different.

8. The Case of a Foreign Monopolist

If there are no government subsidies for the new technology, we have already indicated that the appropriate government benefit function is $Y(m)$ defined by (9), where Y is the domestic net factor income function. The profit maximizing foreign monopolist would choose the markup $\tilde{m} < m^*$ which solves (52) and Proposition 2 tells us $Y(\tilde{m}) \geq Y(m^*)$.

Now let us suppose the government allows a scale subsidy s as in the previous section. We again assume that the monopolist chooses the markup $m(s)$ to solve (53) and that (54) and (55) hold for each s. The government's benefit function is now redefined as follows:

$$B(s) \equiv G(\tilde{m}(s)) - [\tilde{m}(s)+s]z_{K+1}(\tilde{m}(s))$$

$$(57) \qquad \equiv Y(\tilde{m}(s)) - sz_{K+1}(\tilde{m}(s)) \qquad \text{using (14).}$$

Thus the benefit function B in the case of a foreign monopolist is equal to the net output of internationally traded goods G minus monopoly profits and subsidy payments which are assumed to be repatriated abroad.

Proposition 18: Suppose the new technology is held by a foreign monopolist. If the government's initial scale subsidy s is zero, then decreasing s (i.e., imposing a tax) will increase the government's benefit function defined by (57). The government's optimal subsidy is negative.

The above result seems to be a counterpart to the results of Katrak (1977) and Svedberg (1979). They considered models where a multinational enterprise located in a host country behaves like a monopolist. They found that home country welfare could be increased by putting a tariff on the output of the multinational in some circumstances, and thus they independently discovered the proposition that the optimal tariff for a small country might be positive. In our model, the optimal tax or subsidy, $\hat{s}$ say, is obtained by using (57) and solving the equation $B'(\hat{s}) = 0$. However, the level of welfare that the home country could attain if the government set $s = \hat{s}$ would only be optimal relative to the set of instruments we have assumed that the government possesses.[17] If the government could levy a profits tax on industry K+1 or levy a profits tax on remittances sent abroad, then as a limiting case, the government could impose the efficient subsidy rate $\tilde{s} > 0$ defined by (56), the monopolist would be induced to set its markup rate to zero and with a 99 percent profits tax, most of the subsidy profits could be captured by the government, and home welfare and real output would be simultaneously maximized. In fact, under these conditions, the economy could almost attain the competitive post innovation equilibrium described in section 3.

Of course, it is unlikely that a government could impose a sector specific profits tax close to 100 percent for a variety of political and economic reasons.[18] It is also unlikely that the government would have sufficient information to be able to calculate the optimal subsidy $\tilde{s}$ defined by (56), so our analysis in the above paragraph has a slight air of unreality about it.

What may be real about the analysis presented in the last two sections is that modelling the introduction of a closely held new technology into an economy may take on the formal character of a duopoly model, where the government and the monopolist play the roles of the duopolists.[19] We have modelled a few of the possible game theoretic outcomes, but there are many others.

9. Conclusion

The most important results in this paper were Propositions 2 and 4 which may be summarized as follows: a small open economy cannot lose aggregate output if the new technology is adopted, even if all monopoly profits are repatriated abroad. However, Propositions 14 and 15 show that individual competitive domestic inputs[20] may lose as the new technology is introduced.

Some interesting comparative statics results were embodied in Proposition 7, which studied the properties of the domestic net factor income function with respect to the vector of international prices and the vector of domestic factor supplies.

Section 5 derived a number of second order approximations to the home country gain if the new technology were introduced in a competitive fashion. Proposition 10 stated that a quadratic approximation to the gain in factor income from introducing the new technology in a competitive manner is equal to one half of the initial cost advantage times the post innovation competitive scale of the new technology. Our method for proving this Proposition, which involved taking an average of two first order approximations to the gain, should be of wider interest. From the ex ante evaluation point of view, useful second order approximations to the gain were derived in Propositions 12 and 13. These second order approximations to the gain are analogous to the second order approximations to the loss of output due to distortions within the production sector of an open economy derived by Diewert (1983b).[21]

In sections 7 and 8, we considered cases where the new technology was introduced in a noncompetitive manner. We found that the benefits of the new technology will be shared between the monopolist and the host country, no matter whether the monopolist is a domestic resident (see section 7) or foreigner (section 8). We also showed that an omniscient government able to impose sector specific taxes or subsidies on a passive monopolist could increase the gains accruing to the home country (see Propositions 17 and 18). However, we do not regard the results contained in Propositions 17 and 18 as being useful policy prescriptions under normal circumstances for a number of reasons: (i) the government will typically not possess the required information in order to implement an optimal subsidy, (ii) the monopolist may not play a passive role but may behave strategically, and (iii) it is difficult to treat certain sectors in a discriminatory way, both on moral and measurement grounds. Thus in order to maximize the benefits of the new technology to the home country, we would recommend that the home government limit itself to measures that would lead to the eventual competitive introduction of the new technology.

Matematical Appendix

Proof of Proposition 1: The constrained minimization problems (6) and (11) have the same objective functions and the same constraints, except (11) has an additional constraint. Hence for any m, $G^* \leq Y(m)$. By assumption, w^* solves (6). Using (8), w^* is a feasible solution for (11) when $m=m^*$ and thus $Y(m^*) \leq w^*\cdot v = G^*$. Since $G^* \leq Y(m^*)$ as well, $Y(m^*) = G^*$ and w^* solves both constrained minimization problems. Now let $y^1*,\ldots,y^{K*}$, x^{K*} $x^1*,\ldots,x^{K*}$, $z_1*,\ldots,z_K*$ solve (3). Then $y^1*,\ldots,y^{K*}$, $x^1*,\ldots,K*$, $z_1*,\ldots,z_{K*}$ and $z^*_{K+1} \equiv 0$ is a feasible solution for (9) when $m=m^*$ and evaluating the resulting objective function for (9) at this feasible solution yields the value G^*. Since we have shown above that $Y(m^*) \leq G^*$, we see that our feasible solution for (9) when $m = m^*$ is actually an optimal solution.

Proof of Proposition 2: Let $0 \leq m' < m'' \leq m^*$ and let $y^{k''}$, $x^{k''}$, z''_k solve (9) when $m=m''$. Then

$$
\begin{aligned}
Y(m'') &= \textstyle\sum_{k=1}^{K} p\cdot y^{k''} z''_k + (p\cdot y^{K+1} - m'')z''_{K+1} \\
&\leq \textstyle\sum_{k=1}^{K} p\cdot y^{k''} z''_k + (p\cdot y^{K+1} - m')z''_{K+1} && \text{since } z''_{K+1} \geq 0 \text{ and } -m'' < -m' \\
&\leq Y(m') && \text{since } y^{k''}, x^{k''}, z''_k \text{ is feasible for the } Y(m') \text{ maximization problem..}
\end{aligned}
$$

Note that the inequality $Y(m'') \leq Y(m')$ is strict if $z''_{K+1} > 0$.

Proof of Proposition 3: Let $0 \leq m' < m'' \leq m^*$, let $y^{k'}$, $x^{k'}$, z'_k solve (9) when $m=m'$ (and let z'_{K+1} be a maximal z_{K+1} solution so that z'_{K+1} =

$z_{K+1}(m'))$, and let $y^{k''}$, $x^{k''}$, z_k'' solve (9) when $m=m''$ (and let $z_{K+1}'' = z_{K+1}(m''))$. Then

$$Y(m') = \sum_{k=1}^{K} p\cdot y^{k'} z_k' + (p\cdot y^{K+1}-m')z_{K+1}'$$

$$\text{(A1)} \qquad \geq \sum_{k=1}^{K} p\cdot y^{k''} z_k'' + (p\cdot y^{K+1}-m')z_{K+1}''$$

since $y^{k''}$, $x^{k''}$, z_k'' is feasible for the $Y(m')$ problem.

Similary,

$$Y(m'') = \sum_{k=1}^{K} p\cdot y^{k''} z_k'' + (p\cdot y^{K+1}-m'')z_{K+1}''$$

$$\text{(A2)} \qquad \geq \sum_{k=1}^{K} p\cdot y^{k'} z_k' + (p\cdot y^{K+1}-m'')z_{K+1}'$$

since $y^{k'}$, $x^{k'}$, z_k' is feasible for the $Y(m'')$ problem.

Adding the inequalities (A1) and (A2) and cancelling terms yields

$$(m''-m')(z_{K+1}'-z_{K+1}'') \geq 0$$

$$\text{(A3)} \qquad \text{or} \quad z_{K+1}'-z_{K+1}'' \geq 0 \qquad \text{since } m''-m' > 0.$$

The inequality (A3) is strict if either (A1) or (A2) is strict.

<u>Proof of Proposition 4</u>: Upon making the same assumptions as in Proposition 3, we have:

$$G(m') = \sum_{k=1}^{K} p\cdot y^{k'} z_k' + p\cdot y^{K+1} z_{K+1}'$$

$$\geq \sum_{k=1}^{K} p\cdot y^{k''} z_k'' + p\cdot y^{K+1} z_{K+1}''-m'(z_{K+1}''-z_{K+1}')$$

rearranging (A1)

$$= G(m'') - m'(z_{K+1}''-z_{K+1}')$$

$$\text{(A4)} \qquad \geq G(m'') \qquad \text{using (A3) and } m' \geq 0.$$

The inequality (A4) is strict if (A1) is strict or if (A2) is strict and $m' > 0$.

Proof of Proposition 6: We require another expression for $Y(m)$ that can be derived using the Karlin-Uzawa Saddle Point Theorem:

$$\text{(A6)}\quad Y(m) = \max_{z_{K+1}\geq 0}\min_{w\geq 0_N}\{w.(v-x^{K+1}z_{K+1}) + (p\cdot y^{K+1}-m)z_{K+1} : \pi^k(p,w) \leq 0,\quad k=1,\ldots,K\}.$$

Let $0 \leq m' < m'' \leq m^*$, let w' and the maximal z'_{K+1} solve (A6) when $m=m'$, and let w'' and the maximal z''_{K+1} solve (A6) when $m=m''$. Then

$$Y(m') = w'\cdot(v-x^{K+1}z'_{K+1}) + (p\cdot y^{K+1}-m')z'_{K+1}$$

$$\leq w''\cdot(v-x^{K+1}z'_{K+1}) + (p\cdot y^{K+1}-m')z'_{K+1}$$

(A7) since w'' is feasible for the $Y(m')$ max-min problem.

Similarly,

$$Y(m'') = w''\cdot(v-x^{K+1}z''_{K+1}) + (p\cdot y^{K+1}-m'')z''_{K+1}$$

$$\leq w'\cdot(v-x^{K+1}z''_{K+1}) + (p\cdot y^{K+1}-m'')z''_{K+1}$$

(A8) since w' is feasible for the $Y(m'')$ max-min problem.

Adding (A7) and (A8) and cancelling terms yields

$$\text{(A9)}\quad (w'-w'')\cdot x^{K+1}(z'_{K+1}-z''_{K+1}) \geq 0$$

Since $m' < m''$, by Proposition 3, $z'_{K+1}-z''_{K+1} \geq 0$. If any one of the inequalities (A1), (A2), (A7) or (A8) is strict, then $z'_{K+1}-z''_{K+1} > 0$ and (A9) yields the desired result.

Proof of Proposition 7: Let m', p', v', m'', p'', v'' and $0 \leq \lambda \leq 1$ be given. Let y^{k*}, x^{k*}, $z_k{*}$ solve (9) when $m = \lambda m' + (1-\lambda)m''$, $p = \lambda p' + (1-\lambda)p''$, and v is fixed. Then

$$Y(\lambda m'+(1-\lambda)m'', \lambda p'+(1-\lambda)p'',v)$$

$$= \sum_{k=1}^{K}[\lambda p'+(1-\lambda)p'']\cdot y^{k*}z_k^* + [\lambda p'+(1-\lambda)p'']\cdot y^{K+1}z_{K+1}^*-[\lambda m'+(1-\lambda)m'']z_{K+1}^*$$

$$= \lambda[\sum_{k=1}^{K}p'\cdot y^{k*}z_k^*+(p'\cdot y^{K+1}-m')z_{K+1}^*]+(1-\lambda)[\sum_{k=1}^{K}p''\cdot y^{k*}z_k^*+(p''\cdot y^{K+1}-m'')z_{K+1}^*]$$

$$\text{(A10)} \qquad \leq \lambda Y(m',p',v) + (1-\lambda)Y(m'',p'',v)$$

where the inequality follows from the feasibility of y^{k*}, x^{k*}, $z_k{*}$ for the $Y(m',p',v)$ and $Y(m'',p'',v)$ maximization problems.

(A10) establishes the convexity property for Y and we now establish the concavity property. Let v',v'' and $0 \leq \lambda \leq 1$ be given. Let $w*$ solve the minimization problem (11) for some m,p and $v \equiv \lambda v' + (1-\lambda)v''$. Then

$$Y(m,p,\lambda v' + (1-\lambda)v'') = w^*\cdot[\lambda v'+(1-\lambda)v'']$$

$$= \lambda w^*\cdot v' + (1-\lambda)w^*\cdot v''$$

$$\text{(A11)} \qquad \geq \lambda Y(m,p,v') + (1-\lambda)Y(m,p,v'')$$

where the inequality follows from the feasibility of $w*$ for the $Y(m,p,v')$ and $Y(m,p,v'')$ minimization problems of the form (11).

If $v'' > v'$, then using (9), $Y(m,p,v'') \geq Y(m,p,v')$, since the first maximization problem has a bigger feasible region.

To prove the last part, let $m',p',v',y^{k'},x^{k'},z_k'$ be defined as in the statement of the Proposition. For all $p \gg 0_M$ and $0 \leq m \leq m^*$, define the function α by

$$\alpha(p,m) \equiv Y(m,p,v') - [\textstyle\sum_{k=1}^{K} p\cdot y^{k'} z_k' + (p\cdot y^{K+1} - m) z_k']$$

(A12) ≥ 0

where the inequality follows from the feasibility of the $y^{k'}$, $x^{k'}$, z_k' solution for the $Y(m,p,v')$ maximization problem. We also have

(A13) $\alpha(p',m') = 0.$

From (A12) and (A13), the function $\alpha(p,m)$ is globally minimized at p',m'. The first order necessary conditions for a minimum yield

(A14) $\partial\alpha(p',m')/\partial m = \partial Y(m',p',v')/\partial m + z_{K+1}' = 0$ and

(A15) $\nabla_p \alpha(p',m') = \nabla_p Y(m',p',v') - \sum_{k=1}^{K+1} p'\cdot y^{k'} z_k' = 0_M$

(A14) and (A15) may be rearranged to yield (18) and (20).

For all $v \gg 0_N$, define the function β by

$$\beta(v) \equiv Y(m',p',v) - w'\cdot v$$

(A16) ≤ 0

where the inequality follows from the feasibility of w' for the $Y(m',p',v)$ minimization problem defined by (11). By the definition of w',

(A17) $\beta(v') = 0.$

From (A16) and (A17), $\beta(v)$ is globally maximized at v'. Hence

(A18) $\nabla_v \beta(v') = \nabla_v Y(m',p',v') - w' = 0_N$

which may be rearranged to yield (19).

This method of proof is due to Gorman. We note that the differentiability assumptions on Y can be dropped and then the following counterparts to (18)-(20) hold:

(A19) $\quad -z'_{K+1} \varepsilon \partial_m Y(m',p',v')$; $w' \varepsilon \partial_v Y(m',p',v')$; $\sum_{k=1}^{K+1} p \cdot y^{k'} z'_k \varepsilon \partial_p Y(m',p',v')$

where $\partial_m Y$ and $\partial_p Y$ denote the set of subgradients with respect to m and p of the function Y at the point m',p',v' and $\partial_v Y(m',p',v')$ denotes the set of supergradients with respect to v of the function Y at the point m', p', v'. See Rockafellar (1970) for the relevant material on subgradients.

Proof of Proposition 8: From (11), $Y(m) = v \cdot w(m)$ and hence

(A20) $\quad Y'(m) = v \cdot w'(m)$.

Using definition (32), equations (27) may be rewritten as

(A21) $\quad X(m) z(m) = v$

Premultiply the second set of equations in (30) by $z(m)^T$, use (A21) and obtain

(A22) $\quad v \cdot w'(m) = -z(m)^T e_{K+1} = -z_{K+1}(m) \leq 0$ since $z_{K+1}(m) \geq 0$.

(A20) and (A22) yield (36).

Proof of Proposition 9: Using definition (14), for $0 \leq m \leq m^*$,

$$\begin{aligned} G'(m) &= Y'(m) + z_{K+1}(m) + m z'_{K+1}(m) \\ &= -z_{K+1}(m) + z_{K+1}(m) + m z'_{K+1}(m) && \text{using (36)} \\ &= m z'_{K+1}(m) \\ &= -m e_{K+1}^T F(m) e_{K+1} && \text{using (35)} \\ &\leq 0 && \text{since } m \geq 0 \text{ amd } F(m) \text{ is a positive semidefinite matrix.} \end{aligned}$$

Proof of Proposition 10: If Y(m) is a quadratic function of m, then a straightforward computation (see Diewert (1976, p.138) or Denny and Fuss (1983, p.318) shows that the following identity holds:

$$Y(0) - Y(m^*) = (1/2)[Y'(0) + Y'(m^*)][0-m^*]$$

$$\text{(A23)} \qquad = (1/2)Y'(m^*)(0-m^*) - (1/2)Y'(0)(m^*-0)$$

The equality (A23) implies the Proposition. The proof of Proposition 11 is analogous.

Proof of Proposition 14: By using definition (34) and multiplying a matrix times its inverse, we can show that

$$\text{(A24)} \qquad X(m^*)^T E(m^*) = I_{K+1}$$

Hence premultiplying both sides of the equation $w'(m^*) = -E(m^*)e_{K+1}$ (see (35)) yields using (A24),

$$\text{(A25)} \qquad X(m^*)^T w'(m^*) = -e_{K+1}$$

The last equation in (A25) is $\sum_{n=1}^N x_n^{K+1} w_n'(m^*) = -1$ so if $x_n^{K+1} \geq 0$ for each n, then there exists at least one n* such that $-w_{n^*}(m^*) > 0$ which proves (i).

To prove (ii), suppose industry k uses domestic goods as inputs at the pre innovation equilibrium, i.e.,

$$\text{(A26)} \qquad x_n^k(m^*) > 0, \quad n=1,2,\ldots,N.$$

Then the kth equation in (A25) is

$$\text{(A27)} \qquad \sum_{n=1}^N x_n^k(m^*) w'(m^*) = 0.$$

The last equation in (A25) implies $w'(m^*) \neq 0_N$. Hence (A26) and (A27) imply that at least one component of $w'(m^*)$ will be positive and another component will be negative, which proves (ii).

The above method of proof is due to Ethier (1974, p.202) and Jones and Scheinkman (1977, p.926).

<u>Proof of Proposition 15</u>: Since $N = K+1$, (A24) yields

$$\text{(A28)} \qquad E(m^*) = [X(m^*)^T]^{-1}$$

(35), (47) and (A28) yield $-w_1^1(m^*) = -\delta^{-1}x_2^1 \leq 0$ (<0 if $x_2^1>0$) and $-w_2^1(m^*) = \delta^{-1}x_1^1 > 0$. Hence input 1 loses and input 2 gains.

<u>Proof of Proposition 16</u>: From (35), $-w'(m^*) = E(m^*)e_{K+1}$. Since $N = K+1 = 3$, we have (A28). Thus

$$E(m^*) \equiv E \equiv \begin{bmatrix} x_1^1, & x_2^1, & x_3^1 \\ x_1^2, & x_2^2, & x_3^2 \\ x_1^3, & x_2^3, & x_3^3 \end{bmatrix}^{-1} \equiv \begin{bmatrix} A, & b \\ c^T, & x_3^3 \end{bmatrix}^{-1}$$

$$\text{(A29)} \qquad = \begin{bmatrix} A^{-1} + A^{-1}b(x_3^3 - c^TA^{-1}b)^{-1}c^TA^{-1}, & -A^{-1}b(x_3^3 - c^TA^{-1}b)^{-1} \\ -(x_3^3 - c^TA^{-1}b)^{-1}c^TA^{-1}, & (x_3^3 - c^TA^{-1}b)^{-1} \end{bmatrix}.$$

To simplify the notation, define

$$\text{(A30)} \quad \varepsilon \equiv x_3^3 - c^T A^{-1} b = |E|/|A| = \delta^{-1}|E| \neq 0.$$

Thus the sign of ε is equal to the sign of $|E|$ or to the sign of $|X(m^*)|$, the determinant of the 3 by 3 $X(m^*)$ matrix. Using (35), (A29) and (A30), we obtain the following expression for $-w'(0) \equiv -[w_1'(0),w_2'(0),w_3'(0)]^T = Ee_3$:

$$\text{(A31)} \quad -w'(0) = \delta^{-1}\varepsilon^{-1}\ [x_2^1 x_3^2 - x_2^2 x_3^1,\ x_1^2 x_3^1 - x_1^1 x_3^2,\ \delta]^T.$$

If the new industry, industry 3, uses input 3 intensively, so that x^3 is relatively large, then $\varepsilon > 0$ and cases (i) to (vi) follow readily from (A31). If $\varepsilon < 0$, then the signs in cases (i) to (vi) are reversed, yielding cases (viii) to (xii).

Proof of Proposition 17: Replace m by $\tilde{m}(s)$ in (54) and differentiate with respect to s:

$$\text{(A32)} \quad [2z_{K+1}'(\tilde{m}(s)) + (\tilde{m}(s) + s)z_{K+1}''(\tilde{m}(s))]\tilde{m}'(s) = -z_{K+1}'(\tilde{m}(s)) \geq 0$$

where the inequality in (A32) follows from (34) and (35) (remember $F(\tilde{m}(s))$ is a positive semidefinite matrix). (55) and (A32) imply $\tilde{m}'(s) \leq 0$ so if the government increases the subsidy s, the monopolist will generally be induced to decrease his optimal markup.

Proof of Propostion 18: Differentiating (57) and using (A32) yields

$$B'(s) = [Y'(\tilde{m}) - sz_{K+1}'(\tilde{m})]\tilde{m}'(s) - z_{K+1}(\tilde{m})$$

$$\text{(A33)} \quad = [-s(z_{K+1}'(\tilde{m}))^2 + z_{K+1}(\tilde{m})z_{K+1}'(\tilde{m})]/[-2z_{K+1}'(\tilde{m}) - (\tilde{m}+s)z_K''(\tilde{m})]$$

$$-z_{K+1}(\tilde{m}).$$

Using (55), $z'_{K+1}(\vec{m}) \leq 0$ and $z_{K+1}(\vec{m}) \geq 0$, it can be seen that all 3 terms on the right hand side of (A33) are nonpositive if $s \geq 0$. Hence it will not be optimal to have $s \geq 0$. For $s < 0$, the first term is nonpositive and the last two terms are nonnegative. Thus the maximum for $B(s)$ will occur for a negative s.

Footnotes

1. The analysis in this section could be adapted to the problem of measuring gains from eliminating a monopoly in one industry. Thus our differentiable approach has much in common with Harberger's (1974, pp.86-107) differential approach to the measurement of waste due to monopoly.
2. For an excellent summary of the literature with extensions, see Caves (1982).
3. Notation: $p \gg 0_M$ means each component of the M dimensional (column) vector p is positive, $p \geq 0_M$ means each component is nonnegative, $p > 0_M$ means $p \geq 0_M$ and $p \neq 0_M$, and $p \cdot x = p^T x \equiv \sum_{i=1}^{M} p_i x_i$ denotes the inner product of the vectors p and x.
4. For firms that exhibit diminishing returns to scale, we follow the approach of McKenzie (1959, p.66) and introduce a firm specific fixed factor to which the pure profits of the firm will be imputed.
5. Let $(u_1,\ldots,u_M, -v_1,\ldots, -v_N) \in S^k$. If $u_m>0$, then the mth internationally traded good is being produced by sector k while if $u_m<0$, it is being utilized as an input. If $v_n>0$, then the nth domestic good is being used as an input in sector k (this is the normal case), while if $v_n<0$, then good n is being produced as an output by sector k. Thus $(u_1,\ldots,u_M) \equiv u^T$ is a net output vector and $(v_1,\ldots,v_N) \equiv v^T$ is a net input vector (and will normally be nonnegative).
6. See Samuelson (1953), Dixit and Norman (1980) and Woodland (1982).
7. If the mth component of v, v_m say, is negative, then the pre innovation economy is producing the mth domestic good instead of utilizing it as an input.

8. Karlin's version of the theorem requires that: (i) the primal problem have a finite maximum, (ii) the primal objective and constraint functions be concave (and the constraints must be inequalities), (iii) the sets S^k be closed and convex and (iv) the Slater constraint qualification condition hold: i.e., there exist $(\bar{u}^k,-\bar{v}^k)\varepsilon S^k$ such that $\sum_{k=1}^{K}\bar{v}^k \ll v$, an assumption which we now make.

9. See Diewert and Woodland (1977, p.382) for additional details. We require an additional Slater type constraint qualification condition to justify the equality between (5) and (6): i.e., there exists $\bar{w} \geq 0_N$ such that $\pi^k(p,\bar{w}) < 0$ for k=1,...,K. Problem (6) is known as a convex programming problem; see Dantzig (1965, pp.471-481).

10. The complementary slackness conditions (12) imply that each industry that makes negative unit scale profits in equilibrium will operate at 0 scale. Assuming that $z_{K+1}(m) > 0$, (13) implies that the new industry will make exactly the markup m per unit scale.

11. We shall require differentiability at the prices (p,w(0)) and (p,w(m*)) which correspond to the pre and post innovation competitive equilibria.

12. The proof of this statement relies on some rather complex matrix identities developed in Diewert and Woodland (1977) and is omitted.

13. Jones (1979, p.245-246) obtained a similar result in his more complex approach to modelling technological change.

14. The engineering approach to production analysis would play an important role in the determination of the input-output coefficients for the new technology. For references to the engineering approach, see Berndt and Wood (1979), Griffin (1977), and Hoffman and Jorgenson (1977).

15. We may normalize the input-output coefficients of the new technology, $(y^{K+1},-x^{K+1})$, so that at least one component equals unity. Then the

scale variable for sector K+1, z_{K+1}, equals the number of units produced of the good that has the unit input-output coefficient. Then the scale subsidy can be regarded as a subsidy on that good (which is paid only to sector K+1).

16. The government has to know how to compute $z_{K+1}(0)$.

17. An excellent discussion of optimality issues relative to available instrument sets in the context of trade theory is contained in Dixit (1983).

18. For a discussion of this issue and related issues, see Katrak (1981).

19. This game theory framework is particularly relevant if the cost advantage m* of the new technology is "large" so that the intoduction of the new technology will have "large" general equilibrium effects. Examples of large projects which fit into this bargaining framework are the Canadian tar sands and frontier oil exploration projects. In these projects, the Canadian government and multinational oil companies have engaged in very complex bargaining over output, employment and tax policies.

20. If some wages are rigid, then the corresponding factors should be included in the list of "internationally" traded goods.

21. The monopolist's markup m may be regarded as a distortion.

References

Berndt, E.R. and D.O. Wood. "Engineering and Econometric Interpretations of Energy-Capital Complementarity." The American Economic Review 69 (March, 1979): 342-354.

Caves, R.E. Multinational Enterprise and Economic Analysis. Cambridge, England: Cambridge University Press, 1982.

Dantzig, E.B. Linear Programming and Extension. Princeton, N.J.: Princeton University press, 1963.

Debreu, G. Theory of Value, New York: John Wiley, 1959.

Denny, M. and M. Fuss. "A General Approach to Intertemporal and Interspatial Productivity Comparisons." Journal of Econometrics 23 (December, 1983): 315-330.

Diewert, W.E. "Applications of Duality Theory," pp. 106-171 in M.D. Intriligator and D.A. Kendrick (eds.) Frontiers of Quantitative Economics, Vol. II. Amsterdam: North-Holland, 1974.

Diewert, W.E. "Exact and Superlative Index Numbers." Journal of Econometrics 4 (May, 1976): 115-145.

Diewert, W.E. "A Fundamental Matrix Equation of Production Theory with Applications to the Theory of International Trade." Methods of Operations Research 46 (1983): 181-194(a).

Diewert, W.E. "The Measurement of Waste within the Production Sector of an Open Economy." Scandanavian Journal of Economics 85 (May 1983): 159-179(b).

Diewert, W.E. and A.D. Woodland. "Frank Knight's Theorem in Linear Programming Revisted." Econometrica 45 (March, 1977): 375-398.

Dorfman, R., P.A. Samuelson and R.M. Solow. Linear Programming and Economic Analysis. New York: McGraw-Hill, 1958.

Dixit, A. "Tax Policy in Open Economies." Economics Discussion Paper #51, Woodrow Wilson School, Princeton University, Princeton, N.J., 1983.

Dixit, A.K. and V. Norman. Theory of International Trade. Cambridge, England: Cambridge University Press, 1980.

Ethier, W. "Some of the Theorems of International Trade with Many Goods and Factors." Journal of International Economics 4 (May, 1974): 199-206.

Griffin, J.M. "Long-Run Production Modelling with Pseudo Data: Electric Power Generation." Bell Journal of Economics 8 (Spring, 1977): 112-127.

Harberger, A.C. Taxation and Welfare. Boston: Little, Brown and Company, 1974.

Hoffman, K.C. and D.W. Jorgenson. "Economic and Technological Models for Evaluation of Energy Policy." Bell Journal of Economics 8 (Fall, 1977): 444-466.

Jones, R.W. International Trade: Essays in Theory. Amsterdam: North-Holland, 1979.

Jones, R.W. and J.A. Scheinkman. "The Relevance of the Two-Sector Production Model in Trade Theory." Journal of Political Economy 85 (October, 1977): 909-935.

Karlin, S. Mathematical Methods and Theory in Games, Programming and Economics, Vol. 1. Reading Massachusetts: Addison-Wesley Publishing Co., 1959.

Katrak, H. "Multi-National Monopolies and Monopoly Regulation." Oxford Economic Papers 29 (November, 1977): 283-291.

Katrak, H. "Multi-National Firms' Exports and Host Country Commercial Policy." The Economic Journal 91 (June, 1981): 454-465.

McKenzie, L.W. "On the Existence of General Equilibrium for a Competitive Market," Econometrica 27 (March, 1959): 54-71.

Rockafellar, R.T. Convex Analysis. Princeton, N.J.: Princeton University Press, 1970.

Rodriguez, C.A. "Trade in Technological Knowledge and the National Advantage." Journal of Political Economy 83 (February, 1975): 121-135.

Samuelson, P.A. Foundations of Economic Analysis. Cambridge, Massachussetts: Harvard University Press, 1947.

Samuelson, P.A. "Prices of Factors and Goods in General Equilibrium." The Review of Economic Studies 21 (October, 1953): 1-20.

Svedberg, P. "Optimal Tariff Policy on Imports from Multinationals." The Economic Record 55 (February, 1979): 64-67.

Woodland, A.D. International Trade and Resource Allocation. Amsterdam: North-Holland, 1982.

Uzawa, H. "The Kuhn-Tucker Theorem in Concave Programming," pp. 32-27 in Studies in Linear and Non-Linear Programming, K.H. Arrow, L. Hurwicz and H. Usawa (eds.). Stanford California: Stanford University Press, 1958.

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